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DYNAMIC OPERATOR EXTRACTION METHOD

Dymova H. Dynamic Operator Extraction Method. A "black box" is used to mean an object whose internal structure is unknown and information about its structure and functioning can only be partially obtained by analyzing the input-output connections of this object. Not only those material, energy and/or informational flows that are necessary for its functioning in accordance with the goals set before it - signals, but also those that actually complicate the realization of the set goal by the system - obstacles come to the input of the system from the external environment. An unregulated facility is being explored here. When studying such an object, it is important that the signals always describe the behavior of the object as a whole and reflect the individual movements of a large number of its microparticles of the same type. The analysis of the structure of the object based on its established signal is insufficient, if only the dynamic dependence on time is taken into account, even the most detailed registration of the single solution of the established dynamic equation does not allow revealing the structure of the operator in real situations. The inadequacy of the usual black box scheme for studying an unregulated object based on a settled signal leads to the need to account for internal fluctuations in the equations of the object signal. Therefore, the article considers autonomous objects, in the dynamic equations of which time t is not explicitly included. The work formulates and to some extent substantiates a fairly general and fairly simple principle of signal description. According to this basic premise, the properties of the signal, which are quantitatively significant and regularly manifest under the given conditions of observation, are connected to each other by some dynamic structure of the object. The role of object movements, which are less important under these conditions, as well as the role of the external environment, is reflected in this description by the time-fluctuating force $F(t)$ that disturbs the dynamic system. The study of the statistical properties of the response of the dynamic system to the fluctuating disturbance $F(t)$ allows, in a fairly wide range of problems, to evaluate the dynamic characteristics of an unregulated object based on the established signal. The behavior of the signal, which is described by a linearized equation, requires the estimation of the coefficient $A_m^{(k)}(\mathbf{x})$, so the article considers possible schemes for estimating this coefficient.

Keywords: dynamical system, trajectory, fluctuation, phase space, dynamic motion, correlation matrix, mathematical expectation, variance.

Димова Г. О. Метод виділення динамічного оператора. Під "чорною скринькою" прийнято розуміти об'єкт, про внутрішню будову якого нічого невідомо та інформацію про будову та функціонування якого можна частково отримати лише аналізуючи вхідні-вихідні зв'язки цього об'єкта. На вхід системи надходять від зовнішнього середовища не тільки ті матеріальні, енергетичні та/або інформаційні потоки, які необхідні для її функціонування відповідно до поставлених перед нею цілей – сигнали, а й такі, що фактично ускладнюють реалізацію системою поставленої мети – перешкоди. Тут досліджується нерегульований об'єкт. При дослідженні такого об'єкта має значення те, що сигнали завжди описують поведінку об'єкта як цілого і відображують індивідуальні рухи великого числа його однотипних мікрочастин. Аналіз структури об'єкта за його встановленим сигналом недостатній, якщо враховувати тільки динамічну залежність від часу, навіть сама детальна реєстрація єдиного розв'язання динамічного рівняння, що встановилося не дозволяє в реальних ситуаціях розкрити структуру оператора. Непристосованість звичайної схеми чорної скриньки для вивчення нерегульованого об'єкта за сигналом, що встановився, призводить до необхідності обліку внутрішніх флуктуацій в рівняннях сигналу об'єкта. Тому у статті розглядаються автономні об'єкти, в динамічні рівняння яких час t у явному вигляді не входить. В роботі сформульований і в деякій степені обґрунтований доволі загальний та достатньо простий принцип опису сигналу. Згідно цьому основному положенню властивості сигналу, які є кількісно істотними і, що регулярно проявляються при даних умовах спостереження, зв'язуються між собою деякою динамічною структурою об'єкта. Роль менш істотних при цих умовах рухів об'єкта, також як і роль зовнішнього середовища, відображає в цьому описі збурювальна динамічну систему флуктуюча у часі сила $F(t)$. Дослідження статистичних властивостей відгуку динамічної системи на флуктуаційне збурення $F(t)$ дозволяє в доволі широкому колі задач оцінювати за сигналом, що встановився, динамічні характеристики нерегульованого об'єкта. Поведінка сигналу, яка описана лінеаризованим рівнянням, потребує оцінки коефіцієнта $A_m^{(k)}(\mathbf{x})$, тому в статті розглянуті можливі схеми оцінки цього коефіцієнту.

Ключові слова: динамічна система, траєкторія, флуктуація, фазовий простір, динамічний рух, кореляційна матриця, математичне сподівання, дисперсія.

Formulation of the problem. Planning an experiment, solving the problems of identification and forecasting the course of continuous technological processes, the problems of restoring information, as well as the analysis of the results of passive observations are now more important when moving to the study of increasingly complex phenomena. The following sequence of operations corresponds to the traditional research scheme:

1) obtaining experimental data means identifying the qualitative aspects of a phenomenon and building its model, writing model equations and solving them;

2) comparison of the solution with experiments and refinement of the model, etc.

Such a scheme of research is not always expedient. For the study of complex objects, when the overview of measurement results becomes a problem, a "black box" scheme is proposed, in which the effects on the object are described formally as a set of excitations. This scheme, which results from the regular comparison of object responses at the output of the "black box" with excitations at its input, is characterized by a different sequence of operations [1-4]:

1) selection of an a priori class of model equations;
2) comparison of responses with excitations and obtaining specific equations of the model;
3) building a model.

In the simplest version, the relaxation to the steady limit motion in response to the initial deflection is analyzed. In principle, the task of finding the model equations can be solved if the disturbances at the input of the "black box" are sufficiently complete and allow to detect in the responses of the object all its degrees of freedom, essential in the observed phenomenon. Based on the previously formulated and substantiated general principle of signal description, it is necessary to investigate the behavior of the signal, which is described by a linearized equation, and to consider possible schemes for estimating the coefficient $A_m^{(k)}(\mathbf{x})$ of the dynamic system of the object.

Research analysis. The analysis of the dynamic structure of the object refers to three well-known problems of the theory of signals: problems of identification, when on the basis of known signals at the input and output of the system, a conclusion is made about the composition of the system and its characteristics; control problems, when the characteristics of the system and the input signal are known and the law of change of the signal at the output of the system or such an input signal that leads the system to a given state at the output is determined; measurement problems, when the output signal and the characteristics of the system are known, the characteristics of the input signal are determined. The theoretical basis for solving the given problem was the works of the following scientists: H.L. Van Tris, R.E. Kalman, Y.K. Willems, D. Grop.

At the first stage of the work, the analysis was carried out - the selection of a class, where general questions were resolved: based on a priori data about the object under study, one of the types of operator (functional, differential, integral or integral-differential) was reasonably chosen. At the same time, previous information obtained from the signal was taken into account. The statement of the problem and the scheme of its solution in the simplest version of the second stage of the analysis of the structure of the unregulated object are discussed. It was determined that the development of search methods in a certain class of equations belongs to inverse problems of analysis [1].

A general, simple principle of signal description, the properties of which are quantitatively significant and regularly manifested under the given observation conditions, is formulated and substantiated. According to the main provisions, the properties of the signal are connected to each other by some dynamic structure of the object. The study of the statistical properties of the response of the dynamic system to the fluctuating disturbance $F(t)$ makes it possible to evaluate the dynamic characteristics of an unregulated object based on the established signal [2].

Presentation of the main material and justification of the obtained results. Consider one of the possible schemes for estimating the coefficient $A_m^{(k)}(\mathbf{x})$ or $a_k(\mathbf{x})$ of the dynamic system of an object, linearized in deviations from the isolated limit trajectory $\mathbf{U}_0^+(\mathbf{x})$, nowhere in the region $G^+(\mathbf{x})$ filling a densely two-dimensional surface. First, consider the case where $\mathbf{U}_0^+(\mathbf{x})$ does not degenerate into a rest point. Let the record of the signal $U(t;\mathrm{x})$ on the interval $-\frac{\Theta}{2} \le t < \frac{\Theta}{2}$ contain a large number Γ, $(\Gamma \gg 1)$ of stochastically independent realizations $U_\gamma(t;\mathbf{x})(t_{\gamma 1} \le t < t_{\gamma 2}, \gamma = 1, 2, \dots, \Gamma)$. Let us assume for simplicity that each of these realizations at a fixed x falls on the same segment $t'_1(\mathbf{x}) \le t' < t'_2(\mathbf{x})$ of the phase change counted along the trajectory $\mathbf{U}_0^+(\mathbf{x})$ from some point $\mathbf{U}_0(\mathbf{x})$. In such a situation, at a low intensity of internal fluctuations, the observed motions $U_\gamma(t;\mathbf{x})$ repeatedly pass in a fairly narrow neighborhood $G^*(\mathbf{U}_0^+(\mathbf{x})), G^*(\mathbf{U}_0^+(\mathbf{x})) \subseteq G(\mathbf{U}_0^+(\mathbf{x}))$, section (t'_1, t'_2), trajectory $U_0^+(\mathbf{x})$, practically without going beyond G^* for $t'_1 \le t' < t'_2$. This allows on the interval (t'_1, t'_2) to estimate in the space $R_q(\mathbf{x})$ the position of the average statistical trajectory $\mathbf{U}^*(\mathbf{x})$, for which the mathematical expectation of deviations of the ensemble close to $\mathbf{U}_0^+(\mathbf{x})$ trajectories of a fluctuating object. We will calculate starting from the trajectory of some realization $\mathbf{U}_{\gamma 1}(\mathbf{x})$ as from the first approximation: $\mathbf{U}^{(1)}(\mathbf{x}) \equiv \mathbf{U}_{\gamma 1}(\mathbf{x})$. We implement the method of successive approximations as follows: through various points of the trajectory $\mathbf{U}_{(k)}(\mathbf{x})$ of the *k*-th approximation, we draw hyperplanes orthogonal to it. The trajectory passing through the centers of gravity of the points of intersection of these hyperplanes with each of the trajectories $\mathbf{U}_\gamma(\mathbf{x})$ will be taken as the $(k+1)$-th approximation of the average statistical trajectory $\mathbf{U}^{(k+1)}(\mathbf{x})$. The limiting trajectory $\mathbf{U}^{(\infty)}(\mathbf{x})$ for the sequence $\mathbf{U}^{(k)}(\mathbf{x})$ is characterized by the fact that the average value of the deviations of realizations $\mathbf{U}_\gamma(\mathbf{x})$ orthogonal from it vanishes; thus, we can assume that $\mathbf{U}^{(\infty)}(\mathbf{x})$ evaluates the trajectory $\mathbf{U}^*(\mathbf{x})$.

With such a low intensity of internal fluctuations of the object that the observed implementations $G(\mathbf{U}_1(\mathbf{x}))$ do not go beyond the neighborhood $G(\mathbf{U}_0^+(\mathbf{x}))$ [5], where the signal behavior is described by linearized in $(n_1, n_2, \dots, n_{q-1})$ equations

$$\frac{dn_m}{dt'} + \sum_{k=1}^{q-1} A_m^{(k)}(t';\ \mathrm{x}) n_k = F_m(t';\ \mathrm{x}) \quad (m = 1, 2, \dots, q-1),$$

ω-limit trajectory of the dynamic system $\mathbf{U}_0^+(\mathbf{x})$, for which $n_k(t) \equiv 0, k = 1,2, \dots, q-1$, coincides with the average statistical trajectory, counted from $\mathbf{U}_0^+(\mathbf{x})$ orthogonal deviations $n_k^*(t')$ points of which are defined by the formulas $n_k^*(t') \equiv \langle n(t') \rangle$.

Now, knowing the position of the trajectory $\mathbf{U}_0^+(\mathbf{x})$ in the phase space $R_q(\mathbf{x})$, where the Cartesian coordinates are $U, \frac{dU}{dt}, \dots, \frac{d^{q-1}U}{dt^{q-1}}$, it is possible to mark the time of dynamic movement along $\mathbf{U}_0^+(\mathbf{x})$ of the representative point by choosing some state $\mathrm{U}_0^+(\mathrm{x})$ as the initial state. Then, in the neighborhood of $G(\mathbf{U}_0^+(\mathbf{x}))$ for each realization $U_\gamma(t; \mathbf{x})$ we find the functions $n_{k\gamma}(t'; \mathbf{x}), (k = 1,2, \dots, q-1)$ and $\sigma_\gamma(t'; \mathbf{x})$, which, in accordance with the above assumption, are stochastically independent for different γ [6-8].

Let us discuss a calculation scheme that allows us to estimate the values of the coefficients $A_0^{(k)}$ in the equation [9]

$$\frac{d\sigma}{dt'} + \sum_{k=1}^{q-1} A_0^{(k)}(t';\ \mathrm{x}) n_k = F_0(t';\ \mathrm{x}).$$

Let's rewrite this equation for an arbitrary realization of $U_\gamma(t; \mathbf{x})$ in a more convenient form:

$$\frac{d\sigma_\gamma}{dt'}(t' + \tau; \mathbf{x}) + \sum_{k=1}^{q-1} A_0^{(k)}(t' + \tau; \mathbf{x}) n_{k\gamma}(t' + \tau; \mathbf{x}) = F_{0\gamma}(t' + \tau; \mathbf{x}) \qquad (1)$$

$$(\gamma = 1, 2, \dots, \Gamma).$$

Multiplying (1) by $n_{l\gamma}(t'; \mathbf{x}), (l = 1,2, \dots, q-1)$ and introducing the designations

$$\chi_{\Gamma 0 l}^{(t';\mathbf{x})}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t' + \tau; \mathbf{x}) \frac{d\sigma_\gamma}{dt'}(t' + \tau; \mathbf{x}) \qquad (2)$$

$\eta_{\Gamma k l}^{(t';\mathbf{x})}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t'; \mathbf{x})\, n_{k\gamma}(t'; \mathbf{x}),\quad \varphi_{\Gamma 0 l}^{(t;\mathbf{x})}(\tau) \equiv \frac{1}{\Gamma} \sum_{\gamma=1}^{\Gamma} n_{l\gamma}(t'; \mathbf{x})\, F_{0\gamma}(t'; \mathbf{x})$

we get the formulas

$$\chi_{\Gamma 0 l}^{(t';\mathbf{x})}(\tau) + \sum_{k=1}^{q-1} A_0^{(k)}(t' + \tau; \mathbf{x}) \eta_{\Gamma k l}^{(t';\mathbf{x})}(\tau) = \varphi_{\Gamma 0 l}^{(t;\mathbf{x})}(\tau) \quad (l = 1,2, \dots, q-1), \qquad (3)$$

relating the sample correlation functions. As already mentioned, the stochastic connection between the time-shifted values of the fluctuation perturbation F related to the same x disappears at shifts exceeding τ_0. Since the response of a dynamic system to a disturbing force is determined only by the previous values of the force, the stochastic relationship between the signal U, as well as its projections σ and n_l $(l = 1, 2, \dots, q-1)$, on the one hand, and the force F or its projections F_m $(m = 0, 1, \dots, q-1)$ – on the other hand, it disappears when the force response precedes by a time greater than τ_0 [10, 11]. In particular,

$$\langle n_l(t'; \mathbf{x})\, F_m(t'; \mathbf{x}) \rangle = 0,$$
$$\tau \gtrsim \tau_0 \ \ (l = 1,2, \dots, q-1;\ m = 0, 1, \dots, q-1), \qquad (4)$$

this implies

$$\chi_{0 l}^{(t';\mathbf{x})}(\tau) + \sum_{k=1}^{q-1} A_0^{(k)}(t' + \tau; \mathbf{x}) \eta_{k l}^{(t';\mathbf{x})}(\tau) = 0, \tau \geq \tau_0 \ (l = 1, 2, \dots, q-1), \qquad (5)$$

where

$$\chi_{0l}^{(t';\mathbf{x})}(\tau) \equiv \langle n_l(t'+\tau;\mathbf{x}) \frac{d\sigma}{dt'}(t';\mathbf{x})\rangle,$$

$$\eta_{lk}^{(t';\mathbf{x})}(\tau) \equiv \langle n_l(t'+\tau;\mathbf{x}) n_k(t';\mathbf{x})\rangle.$$

Since, in addition,

$$\langle \left[\varphi_{\Gamma 0l}^{(t;\mathbf{x})}(\tau)\right]^2 \rangle = \frac{1}{\Gamma}\langle [n_l(t';\mathbf{x})\, F_0(t'+\tau;\mathbf{x})]^2 \rangle, \quad (6)$$

can be estimated in the zeroth approximation the coefficients $A_0^{(k)}(t';\mathbf{x})$ by the values $A_0^k(t';\tau;\mathbf{x})$, based on the approximate formulas

$$\chi_{0l}^{(t'-\tau;\,\mathbf{x})}(\tau) + \sum_{k=1}^{q-1} \mathcal{A}_0^{(k)}(t';\mathbf{x};\tau)\eta_{kl}^{(t'-\tau;\,\mathbf{x})}(\tau) = 0, \quad (7)$$

where, as τ we take the shift lying inside the interval $\tau_0 \leq \tau < \Theta_m(\mathbf{x})$.

Indeed, the random variables $\varphi_{\Gamma 0l}^{(t';\mathbf{x})}(\tau)$, which we neglected in the right-hand side of equations (3) after writing (7), have at $\tau \geqslant \tau_0$ according to (5), (6) zero mathematical expectations and variances tending to zero with increasing Γ [3, 4]. The results of such an estimate are random functions of τ.

In order to make fuller use of the information contained in the observed signal, it is natural to refer to the estimates of the coefficients $A_0^{(k)}$, based on the minimum of the root-mean-square error formed by the values $\varphi_{\Gamma 0l}^{(t';\mathbf{x})}(\tau)$. However, the direct use of the technique usually used in such estimates to equations (3) at $t'+\tau = \Theta$ and different τ in order to obtain relations relating, like (6), the coefficients for identical phases (Θ), is hindered by a significant stochastic relationship between values $\varphi_{\Gamma 0l}^{(t';\mathbf{x})}(\tau)$. Indeed,

$$\langle \varphi_{\Gamma 0l}^{(\Theta-\tau_1;\mathbf{x})}(\tau_1)\, \varphi_{\Gamma 0k}^{(\Theta-\tau_2;\mathbf{x})}(\tau_2)\rangle =$$
$$= \frac{1}{\Gamma}\langle n_l(\Theta-\tau_1;\mathbf{x})\, n_k(\Theta-\tau_2;\mathbf{x})\, [F_0(\Theta;\mathbf{x})]^2\rangle.$$

For large Γ, the distribution of the vector random process $\varphi_{\Gamma 0l}^{(\Theta-\tau;\mathbf{x})}(\tau)$ $(l = 1, 2, \ldots, q-1)$ tends to the Gaussian law; taking this into account, for $\tau_1, \tau_2 \geqslant \tau_0$ we can rewrite the last expression for its correlation matrix in the form

$$\langle \varphi_{\Gamma 0l}^{(\Theta-\tau_1;\mathbf{x})}(\tau_1)\, \varphi_{\Gamma 0k}^{(\Theta-\tau_2;\mathbf{x})}(\tau_2)\rangle =$$
$$= \frac{1}{\Gamma}\langle [F_0(\Theta;\mathbf{x})]^2\rangle \langle n_l(\Theta-\tau_1;\mathbf{x})\, n_k(\Theta-\tau_2;\mathbf{x})\rangle. \quad (23)$$

Conclusions and prospects for further research. For almost identical vanishing of all components of matrix (8) and, accordingly, disappearance at large Γ of the stochastic connection between the values $\varphi_{\Gamma 0l}^{(\Theta-\tau;\mathbf{x})}(\tau)$ such large shifts are needed that the matrix $\eta_{kl}^{(\Theta-\tau_1;\mathbf{x})}(\tau-\tau_1)\mathbf{x}$. But with such shifts, as it can be seen from equation (3), it is unreasonable to estimate the coefficients A_0^k while smaller values of $|\tau_1 - \tau_2|$ correspond to the components of the correlation matrix (8), falling off as $1/\Gamma$, that is, according to (5), at the same rate as the dispersions $\varphi_{\Gamma 0l}^{(\Theta-\tau;\mathbf{x})}(\tau)$.

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