THE MONODROMY MATRIX CONSTRUCTION FOR EXECUTIVE OBJECT OF A NONLINEAR SYSTEM.

Kostiuchko S.M., Kyryliuk L.M., Kalys O.V., Sibanda Z.F., Havryliuk S.A. The monodromy matrix construction for executive object of a nonlinear system. The article reveals one of a monodromy matrix constructing methods, reveals the essence of the simplest construction and calculation such matrix. This method is used to build a actuator mathematical model, which can be used to study transients and steady-state processes.

Keywords: monodromy matrix, mathematical model, asynchronous motor, state equation.

Problem formulation

The monodromy matrix of any physical device is used in the steady-state processes, static stability and parametric sensitivity analysis. We will show how to find it in the case of electrical devices using mathematical models on the example of an induction motor with a rotor rectangular groove.

Recent research analysis

To solve this problem, it was necessary to solve several important theoretical problems, in particular, to develop: a general theory of the electric skin effect, the principle of constructing circuit-field electrical devices mathematical models, and the theory of parametric sensitivity auxiliary model. This became the basis for constructing matrices of monodromy systems.

Mathematical model

The machines rotor winding by the number of turns is considered to be reduced to 125 stator windings. Thus, it is flowed not by physical but by reduced currents. At the same time, these currents appear in the boundary conditions of the electromagnetic field. Therefore, the equation of the electromagnetic circuit also must be reduced together with the currents. For which it is enough to give the environment parameters. The rotor is made in the squirrel wheel form. This is a multi-loop system, which is usually equivalent to two circuits. However, such equivalence should not affect the geometry of the groove space. All groove dimensions must be intact. In the transformed coordinates, the rotor winding currents and voltages also are reduced in frequency to the currents and voltages of the stator winding. The electromagnetic field equation also must be given. The motor's electromagnetic state equation is written in the form

\[
\frac{d\hat{A}}{dt} = A(u - \Omega'\Psi - R_i) \tag{1}
\]

where
Here $i = (i_A, i_B)$, $k = S, R$ are columns of stator winding phase currents and the rotor winding converted currents; $u = (u_A, u_R)$, $k = S, R$ are the stator winding phase voltages columns; $A_S, A_{SR}, A_R, A_k$ are matrices $A_S = a_S(1 - a_S G); A_{SR} = A_{RS} = -a_S a_R G; A_R = a_R(1 - a_R G)$.

where $G, \Omega$ are matrices

$$G = \begin{bmatrix} T + b_A i_A & b_B \\ b_A i_B & T + b_B i_B \end{bmatrix}, \Omega = \begin{bmatrix} \omega & -1 \\ \sqrt{3} & 0 \end{bmatrix}.$$  

moreover

$$b_A = b(2i_A + i_B); b_B = b(i_A + 2i_B); b = \frac{2R - T}{3i_m^2};$$

$$R = \frac{1}{a_S + a_S + \rho}; T = \frac{1}{a_S + a_S + \tau}. $$

Here $\tau, \rho$ are the characteristic of magnetization (idling) of the machine as:

$$\tau = \begin{bmatrix} \psi_m(i_m) \\ i_m \end{bmatrix}^{-1}; \rho = \begin{bmatrix} d\psi_m(i_m) \\ di_m \end{bmatrix}^{-1},$$

where $i_m$ is the module of spatial vector of magnetization currents

$$i_m = 2\sqrt{(i_A^2 + i_A i_B^2 + i_B^2)^2}; i_A = i_{SA} + i_{RA}; i_B = i_{SB} + i_{RB}. $$

In the absence of saturation, the magnetization characteristic degenerates directly $i_m = a_m \psi_m$, where $a_m$ is the main inductedance, and the matrix (4) according to (6) degenerates into a diagonal

$$G = \begin{bmatrix} 1 \\ a_S + a_R + a_m \end{bmatrix}.$$  

which greatly simplifies the equation (1). In this case, we obtain the simplest of all known the induction motor mathematical models. $R_S, R_R$ are resistance matrix

$$R_S = \begin{bmatrix} r_S \\ r_S \end{bmatrix}; R_R = \begin{bmatrix} r_R \\ r_R \end{bmatrix}, $$

moreover $a_S, a_R$ are inverse inductances of dissipation of stator and rotor windings; $r_S$ is stator phase resistance; $r_R$ is the resistance of the rotor winding; $\Omega$ are angular velocity matrix $\omega$.

The column's components of complete flux couplings of the stator and rotor windings are found as follows

$$\Psi_{kj} = \frac{1}{\tau} (i_{Sj} + i_{Rj}) + \frac{1}{\alpha_k} i_{kj}, j = A, B; k = S, R.$$  

Column's elements of a stator and rotor voltages

$$u_S = (U_m \sin(\omega_0 t), U_m \sin(\omega_0 t - 120^0)); u_R = 0,$$

where $U_m$, $\omega_0$ are amplitude and circular frequency of network voltage.

The equation of mechanical state has the form

$$\frac{d\omega}{dt} = J(M_e - M) / p_0, $$

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moreover

\[ M_E = \sqrt{3} p_0 (\Psi_s i_S - \Psi_{sb} i_{SA}), \]

where \( M(\omega) \) is mechanical moment; \( p_0 \) is the magnetic poles number of pairs; \( J \) is the moment of rotor inertia; \( M_E \) is electromagnetic moment.

System of differential equations (1), (12) are mathematical A-model of the actuator (asynchronous motor). It is intended for the transient and steady-state processes analysis. For its practical use it is necessary to know the following object parameters: stator and rotor windings dissipation supports and inverse inductances; idle characteristic, and without taking into account the saturation of the main magnetic circuit - the machine's inverse main inductance, the magnetic poles number of pairs and the rotor inertia moment. The input signals are: phase supply voltages and mechanical torque on the shaft.

**Monodromy matrix construction**

When constructing an auxiliary sensitivity model, we form a unknowns column \( y(t) = (\Psi, \omega), \)

\[ y(t) = (\Psi, \omega), \] (14)

The monodromy matrix is written in the form (everywhere later the matrix \( S = z \))

\[ \Phi = (Az, w), \] (15)

Thus, the executive object’s monodromy matrix construction to requires the integration the first variation's equations \( \frac{dz}{dt} \) and \( \frac{dw}{dt} \):

\[ \frac{dz}{dt} = -(\Omega + RA)z - \frac{\partial \Omega}{\partial \omega} w \Psi, \] (16)

where

\[ z = \frac{\partial \Psi}{\partial x(0)}; \quad w = \frac{\partial \omega}{\partial x(0)}. \] (17)

Matrix \( \frac{\partial \Omega}{\partial \omega} = \text{const}. \) The matrix structural form can be determined using the equation \( \Omega = \text{diag}(1, \Omega_R) \) and formula

\[ \Omega = \frac{\omega}{\sqrt{3}} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}. \] (18)

\[ \frac{dw}{dt} = \frac{p_0}{J} \left( \sqrt{3} p_0 \left( \frac{\partial \Psi_s}{\partial x(0)} i_s + \Psi_s \times \frac{\partial i_s}{\partial x(0)} \right) - \frac{\partial M(\omega)}{\partial \omega} w \right). \] (19)

On the s-th iteration of Newton's formula

\[ x(0)^{(i+1)} = x(0)^{(i)} - f'(x(0)^{(i)})^{-1} f(x(0)^{(i)}), \] (20)

linear variational equations (16), (19) are subject to compatible integration with nonlinear (1), (12) on the time interval \([0, T]\).

The proposed analysis method has received a comprehensive test in complex problems of electromechanics. And it turned out to be very effective.

**Conclusions**

Determination of the electrical devices monodromy matrix is most simply carried out on the basis of integrating the first variation equations of the differential equations of device's state. Only on the basis of the monodromy matrix there is a practical possibility to build general algorithms for the analysis of physical devices in full, using the general theory similar mathematical apparatus of nonlinear differential equations. This applies to the analysis of transients and steady-state processes, determining the static stability of the found steady-state processes and, finally, to find parametric sensitivities matrices in transient and steady-state processes.
References


